An inquiry concerning common misconceptions about randomness

Abstract

Randomness is a central concept in everyday life as well as in statistics and physics. Still, there is no consensus about how to formulate a definition. To clarify the characteristics of randomness, a number of misconceptions are identified and analysed. As a consequence of this investigation, randomness is described as partly ordered and as a crucial element in time-dircected processes of growing structure and regularity. Furthermore, a prejudice that implies that randomness is a requisite for fair games of chance is scrutinised and found to be false. The most important result of these clarifications is that they imply the existence of an additional physical law that describes the occurrence of the special kind or irregularity called randomness.

1 Introduction

In daily life, the word "random" is often used to represent events that are unexpected and seemingly without order. In science, on the other hand, randomness is connected with deeply problematic issues of physical and philosophical nature. There are still doubts whether the equations of quantum physics describe fundamentally random processes (Aaronson, 2014), and the statisticians have a hard time to find out whether probabilities ought to be applied according to frequency, bayesian or subjective interpretations (Gillies, 2000; Greco, 2011).

The lack of consensus about these issues is to some extent due to disagreement about definitions of basic concepts. The aim of the present essay is to clarify the meaning of the randomness concept through analyses of some common misunderstandings regarding this subject. In order to narrow the analysis, the type of randomness and probability described here will only concern events that are possible to repeat under similar conditions. Consequently, randomness and probability that describe unique events are not considered, and the concept of subjective probability will not be scrutinised.

In section 2 a definition of what is here meant by randomness is provided. In section 3 the belief that randomness represents maximal disorder is proven to be false. In section 4 this result is used as a basis for a description of how randomness, in contrast to what is often presumed, contributes to order and the growth of structure in our world. In section 5 it is shown that randomness, which is rarely assumed to have a preferred direction in space and time, is connected with future-directed probabilities rather than with past-directed probabilities. In section 6 it is demonstrated that randomness is not a necessary condition for chance in a fair lottery or in any other game of chance with a finite number of outcomes and where not all the players are collaborating. Finally, in sections 7 the implications of these results are discussed.

2 Definition of randomness

There is no generally accepted definition of randomness. According to Wikipedia (2023), the term is commonly used to denote events that lack pattern or predictability. However, such a formulation reveals contradictory features of the commonplace meaning of randomness.

If a large number of random events demonstrate a total absence of pattern with respect to the individual outcomes, the proportions of these outcomes are expected to stabilise close to certain numbers, which represent the probabilities of the outcomes. For example, if somebody flips a fair coin in a way that does not produce any patterns, the proportion of heads can be predicted to be closer to $\frac{1}{2}$, the greater the number of flips grows. Thus, the short-term lack of pattern is connected with a kind of long-term outcome that contradicts the asserted lack of predictability.

Furthermore, if the randomness is characterised by a lack of predictability (rather than lack of "retrodictability", i.e., the ability to say or estimate what happened in the past), the concept has an asymmetry with respect to time. Thus, the lack of predictability is connected with a direction in time that can be considered as kind of pattern.

Consequently, a randomness concept based on a general lack of pattern and predictability seems to be problematic. A scientific definition of randomness ought to be built on clear concepts that do not lead to contradictions, and such one was given by the mathematician von Mises:

> In this way we arrive at the following definition [of randomness]: A collective appropriate for the application of the theory of probability must fulfill two conditions. First, the relative frequencies of the attributes must possess limiting values. Second, these limiting values must remain the same in all partial sequences which may be selected from the original one in an arbitrary way. (von Mises, 1957, p. 24)

Hence, this definition is grounded on what von Mises (1957) called the *Law of Stability of Statistical Frequencies* and the *Principle of the Impossibility of Gambling Systems*.

Another, and less strict, definition of randomness was formulated by the philosopher Mario Bunge:

> ... the particular kind of disorder characterized by local irregularity (e.g., individual coin tossing) combined with global regularity (e.g., long-run equal chances of heads and tails). (Bunge, 1999, p. 83)

These two definitions harmonise well and will serve as points of departure for the present essay. It should be noted that none of them refers to any kind of time-directed "predictability", and that the lack of pattern is restricted to the individual ("local") events and not to the long-term ("global") series of events.

3 Randomness as partly ordered

According to he definition of Bunge (1999) in section 2, randomness is characterised by irregularity on the local but not on the global level. The reason for this characterisation is that a random sequence of events that happen under similar conditions is expected to be ruled by the aforementioned *Law of Stability of Statistical Frequencies.* Thus, such series of events have a kind of large-scale order, where the proportions of different outcomes will agree with the probabilities of these outcomes with a high degree of certainty.

This orderliness is in conflict with a common prejudice according to which randomness is an extreme form of disorder. The prejudice would be further refuted, if it could be demonstrated that there are classes of events that represent even more irregular forms of disorder than randomness. This can be done in several processes, for example in the following simple experiment.

Assume that somebody throughs an irregular stone that can land in a finite number of different stable positions on a surface that is horisontal and plane (the position is defined by which parts of the stone that is in contact with the surface). If this event is repeated many times under similar conditions, it would be expected that the frequencies of the possible positions will stabilise in accordance with some unknown probabilities. Presumably, every skilled statistician or physicist would be very sure that such predicted stabilising outcome will be realised, regardless of the specific characteristics of the stone.

Furthermore, assume the existence of another universe, where the known physical laws are exactly the same as in our universe, but where the outcomes of the trials with the same stone do *not* stabilise in accordance with some unknown probabilities. As a suggestion, when experiments in that alternative universe are performed in the evening the outcomes stabilise around different frequencies than they do in the morning, and if the stone is thrown at different places, the frequencies will differ again. This would be a kind of outcome that is even more disordered than the one connected with randomness.

An interesting question is whether such a universe is possible, i.e., if it is possible that a universe with exactly the same known physical laws as ours could exhibit different statistical outcomes in such a simple experiment. If the answer of this question affirmative, it has a far-reaching consequence. In two universes with identical physical laws, identical experiments must have outcomes that are in accordance with these laws.[1] Consequently, if the *known* laws in the alternative universe is the same as in ours, and if an identical experiment in that universe has another outcome than in ours, there must be some physical law or laws missing in our descriptions of the two universes.

So how can it be examined if the known physical laws of our universe allow a disordered and repeatable phenomena to evolve without any stabilising relative frequencies? Since the physical laws in our universe are time reversal invariant and therefore are applicable in both time directions (except the second law of thermodynamics, which cannot be expected to have any relevance for the statistical outcome of the experiment with the stone), this can be done by performing an experiment similar to the throwing of the stone but in the reverse time order.

Assume that the stone, instead of being thrown, is shaken inside a box (without lid) that is turned upside down. If the box is large enough to allow the stone to take every possible position inside it, the outcome *after* this procedure can be expected to show the same kind of stabilising frequencies as if the stone had been thrown. On the other hand, the position of the stone *before* the box was shaken does not necessarily exhibit stabilising frequencies. An information-processing creature (e.g., a human being) or a machine can put the stone in an initial position according to some system without any stablising frequencies (a similar experiment with a die inside a box is analysed in Skoruppa (2022c, section 4).

Hence, the very same procedure – a stone shaken under a box – makes us sure that the outcomes will exhibit stabilising frequencies *after* it has taken place but not *before*. Since the physical laws in our universe

[1] In accordance with von Mises, the laws of statistics are assumed to be universal on the same footing as conventional physical laws:

> Like all the other natural sciences, the theory of probability starts from observations, orders them, classifies them, derives from them certain basic concepts and laws and, finally, by means of the usual and universally applicable logic, draws conclusions which can be tested by comparison with experimental results. In other words, in our view the theory of probability is a normal science, distinguished by a special subject and not by a special method of reasoning. (von Mises, 1957, p.s 31)

are time reversal invariant, the past-directed evolution before the stone-shaking procedure demonstrates that our physical laws *allow* an outcome of the procedure without any stabilising frequencies. Given that the stone is shaken under the box, the irregularity of the outcomes in the future direction are in accordance with the definitions of randomness in section 2, but in the past direction they are even more irregular.

Moreover, according to the reasoning above, this reveals that the laws of our universe are incomplete in this respect. The difference between the statistical outcomes in the future and past time-directions in connection with the stone-shaking procedure calls for an explanation in the form of some kind of physical law or laws that describes this difference.[2]

It may seem as a very pretentious claim that a limited experiment with a stone, shaken under a box, proves that there are events in our universe that are more disordered than random events and, moreover, to assert that this calls for an explanation in the form of a additional physical law or principle. However, the same kind of total lack of disorder can be demonstrated on a more fundamental level in the realm of quantum mechanics.

The physicist Roger Penrose (1989) has described an illustrative example of the future-directed randomness and past-directed total lack of order and regularity in a simple thought experiment where a photon meets a half-silvered mirror (Figure 1). If a large number of experiments are carried out in the future time direction, the frequencies will stabilise with about 50 percent of the photons passing through the mirror while the rest of them will be reflected.

In the past time direction no such orderliness can be expected. The number of photons that has been reflected by or has passed through the mirror M, given that they have been detected at P, can differ from time to time and from place to place depending on the conditions under which the experiment is performed. In correspondence with the irregular past of the stone-shaking experiment, you cannot give any law-like probabilities that describe whether the photon detected at P has its origin in B or in L.

For example, if there is a light source also at B, there will be a different result than if there is only a light source at A, and those two sources can be combined in arbitrary ways, all of which have the result

[2] It could be tempting to try to explain the lack of randomness in the past time-direction of the stone-shaking procedure as a result of the human capacity to prepare but not to "postpare" (i.e. to decide the future conditions of) an indeterministic experiment. However, the act of preparation cannot be distinguished from the act of "postparation" in the realm of physics, since it usually refers to metaphysical entities such as "free will" and "universal causation". As Sklar (1986, p. 220) puts it, "it is a slippery and difficult notion". For further details about this issue, see (Skoruppa, 2022c, section 8).

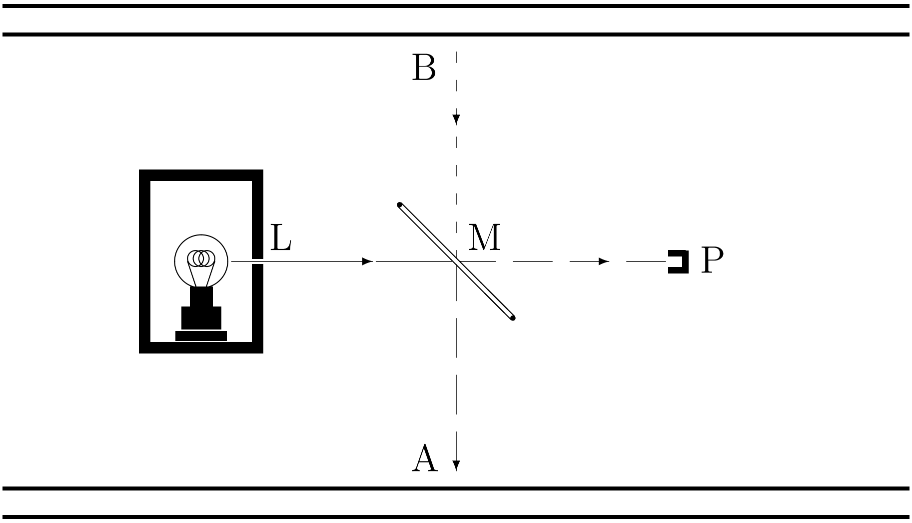

Figure 1: The half-silvered mirror.

that photons with different past histories are detected at P. In a similar way, a lot of examples can be found in both microscopic and macroscopic events that confirm that when randomness, as defined in section 2, reigns, there is a larger degree of disorder in the past time direction than in the future time direction.[3]

In conclusion, the kind of disorder that is demonstrated in the case of randomness is not a maximal kind of disorder, and more disorderly phenomena are common in our universe. Furthermore, this reveals that there must be some missing physical law or laws that describe in which cases randomness should be expected to occur.

4 Randomness as a source of structure

On planet earth, the most striking demonstration of the creative role of randomness seems to appear in the evolution of species, as it was originally described by Darwin (1859). The complex structures of living creatures calls for an explanation that for most of the human history has been sought in terms of some form of deity or a creator. However, in the recent two centuries it has become evident for most people that the Darwinian explanation is the correct one.

The basic mechanism behind evolution is the influence from random cosmic radiation that causes mutations on the chromosomes. These mutations modify, in their turn, the characteristics of the organisms, which the chromosomes contain information about.

[3]For more examples, see Skoruppa (2022b,c).

To be efficient, evolution also need to consist of an orderly part that works in tandem with the disorderly part represented by the mutations. This orderly part is called *natural selection* and functions through the capacity of a mutated individual to survive long enough to reproduce and pass the mutation to the next generation. Most of the time, the modified chromosomes have characteristics that do not improve the possibility of the organism to survive, but sometimes they do. As a result, successful randomly caused mutations are selected for the next generation.

This combination of randomness and a selective process seems to be a common feature of many sorts of creative processes. A music composing computer program, *Iamus*, which creates original pieces of music, and which subsequently has been performed by professional orchestra, uses both random components and selective structuring rules (Diaz-Jerez, 2011). Campbell (1960) describes how humans need both seemingly random generated impulses and a critical mind to invent new and creative ideas. Simonton (2011) writes in an article based on the work of Campbell:

> Despite the above precaution, it remains true that the creative process often operates *as if* it were generating random variants. That is probably why, as mentioned earlier, computer programs that most successfully simulate or exhibit creativity tend to rely on some stochastic mechanism.

On a more fundamental physical level, the most dominant growth of structure and order is demonstrated by the emergence of stars, planets, black holes, galaxies and galaxy clusters as a result of the seemingly totally disordered Big Bang. Here, random quantum fluctuations is hypothesised to have formed a basis for an evolution, where small "seeds" of mass aggregation grew to formations of greater complexity with time. The selective part or the process is represented by the *gravitational force*, which has a gathering tendency that works in tandem with the randomly spreading tendency of the Big Bang (Guth & Kaiser, 2005).

Another broad field where randomness can be expected to contribute to order is in *self-organising systems*. Unfortunately, the mechanisms behind these systems are far from understood.[4] However, some ideas about their time asymmetry and dependence of randomness are put forward in (Skoruppa, 2022d, chapter 20).

It should be noted, of course, that randomness is far from being a guarantee for order and structure when it appears in isolation, for example in a game of chance or in a disorderly process of statistical

[4] Banzhaf (2009) writes: "Despite half a century of inquiry, the theory of self-organizing systems [SOS] is still in its infancy. There is no 'standard model' of SOS, only various aspects emphasized by different researchers."

mechanics. In such processes, randomness often leads to unpredictability and to dispersion of matter.

In conclusion, randomness can be a source of order and *future-directed* growth of structures, if it works in tandem with some kind of principle that contributes with order and regularity.[5] This can be considered as further confirmation for the conclusion from section 3 that randomness does not represent maximal disorder.

5 Randomness as a time-directed phenomenon

In section 3, it was revealed that randomness characterises some processes in the *forward* time direction. A stone or a dice that is shaken inside a box displays randomness according to the definitions in section 2 *after* the shaking procedure but not necessarily before the procedure. A photon that is sent through a half-silvered mirror gives outcomes characterised by randomness if detected *after* the interaction with the mirror, not before. In section 4 another kind of time-direction connected with randomness was found in the growth of structure in the *future* time direction. The question is whether these time asymmetries of randomness are signs of a more general principle.

It is noteworthy that *The origin of species ...* written by Charles Darwin (1859) only contains one illustration, and that this illustration has a clear time asymmetry. It depicts the evolution of the species through time, and it takes the form of a tree, which branches off in the *future time direction* each time a new species becomes different enough from its neighbouring relatives to be defined as unique.

An interesting feature of the kind of evolutionary randomness that was discussed in section 4 is its time-directed character. Assume that two human parents, which both have a gene for blue eyes and a gene for brown eyes, get three children. Then, the *future-directed* probability for each child to have brown eyes is about 0.75, irrespective of where and when the children are born, since the gene for brown eyes is dominant.[6] On the other hand, if we know that two parents have three children with brown eyes, we cannot tell the *past-directed* probability that both parents have a gene for blue eyes and a gene for blue eyes. This follows from the fact that the past-directed probabilities depend on the number

[5] It should be noted, of course, that randomness is far from being a guarantee for order and structure when it appears in isolation, for example in a game of chance or in a disorderly process of statistical mechanics. In such processes, randomness often leads unpredictability and to dispersion of matter.

[6] Future-directed probability is defined as a conditional probability where the conditions precedes the outcome, and past-directed probability is defined as a conditional probability where the outcome precedes the condition. The physical role of these probabilities are described in Skoruppa (2022a,b).

of parents with blue eye genes and brown eye genes in the population as a whole, while the future-directed probabilities are independent of these numbers.

It should be noted that any alleged dominance of the future-directed probabilities can never be a question about whether there exist such probabilities to a greater extent than there exist past-directed probabilities. Every indeterministic event can be described in terms of both types of conditional time-directed probabilities, depending on which state is described as the condition and which state is described as the outcome. The point is that the future-directed (but not necessarily the past-directed) probabilities are characterised by randomness. According to the definitions in section 2, this means that the outcomes have a law-like character, since they are ruled by the *Law of Stability of Statistical Frequencies* and the *Principle of the Impossibility of Gambling Systems*.

Another way to express the law-like character of the future-directed probabilities is to state that they are *translation invariant in time and space*, which means that they remain the same from time to time and at different places. This constraint follows from the fact that, according to the definitions in section 2, events characterised by randomness must be possible to repeat with constant probabilities, represented by "limiting values [that] must remain the same in all partial sequences which may be selected from the original one in an arbitrary way". This is possible only if the future-directed probabilities are equal irrespective of place and point in time, i.e., it they are translation invariant in time and space.

To simplify the issue even more, translation invariance in time concerning probabilities that describes physical processes always, as a convention, tacitly implies translation invariance in space as well. Hence, the assumption of time translation invariance is an alternative way to assume that the events in question are characterised by randomness in accordance with the definitions in section 2.[7]

Thus, the time directed characteristic of randomness can be expressed as a physical law, which was recently proposed by Skoruppa (2022b,c):

> **Law of general statistical time asymmetry**
> Assume that $A[t_1, t_2]$ and $B[t_3, t_4]$, where $t_1 \leq t_2 \leq t_3 \leq t_4$ are physical states, measured by the same quantities in a process that is uncontrolled in the time interval $[t_1, t_4]$,

[7]The relation between time translation invariance in space and time, on the one hand, and the statistical laws represented in Section 2, on the other hand, is to some degree described in (Skoruppa, 2022d, chapter 8, section "Fysisk symmetri"), which, unfortunately, is written in Swedish. The interrelations between these concepts are in need to be further explored, but this will not be done in the present essay.

> and where only one of the non-deterministic probabilities $p(B[t_3, t_4]|A[t_1, t_2])$ and $p(A[t_1, t_2]|B[t_3, t_4])$ is time translation invariant. Then the former probability is always time translation invariant.

The word "uncontrolled" here means that no manipulation is made during the process to control the outcome of the process in either time direction, which is a more loose condition than isolation. The validness of this law is tentatively confirmed in the Skoruppa (2022d) and in the appendices published in connection with that book.

In conclusion, randomness has a direction in time that contrasts with the prejudice that this phenomenon lacks any kind of direction in space and time.

6 Chance without randomness

According to another common misconception about randomness, it is a requisite for fair outcomes in games of chance. In this section it will be demonstrated that this is not the case in the simplest possible of such games, i.e. lotteries with one single winner. This result can be generalised to other games of chance, given that they have certain characteristics.

Assume that n individuals want to conduct a fair lottery, where one of the participants will get a prize at the end of the procedure. As they receive their ticket, they are informed about their unique and arbitrarily chosen ticket number from 1 to n. In return, each participant delivers a freely chosen number between zero and n to the administrator of the lottery. The administrator then sums the individually given numbers and consecutively subtracts n from the result until a number less than n remains. This number plus one constitutes the winning number.[8]

As an example, assume that five participants receive a ticket each, and that they deliver a number to the administrator in accordance with the following scheme, where the numbers inside the brackets denote {ticket number, delivered number}: {1,3}, {2,5}, {3,1}, {4,5}, {5,2}. The sum of the individually delivered numbers is $3 + 5 + 1 + 5 + 2 = 16$. Since $16 - 3 \cdot 5 + 1 = 2$, the owner of ticket number 2 is the winner.

This procedure can be generalised to any game of chance, given that the game has a finite number of outcomes and given that not all the players are collaborating (this condition hinders collaboration between all the players against the bank, which can result in ordered outcomes). For example, if an arbitrary number of players want to mimic the random

[8] In mathematical language this means that the winning number is $\text{mod}_n(s) + 1$, where s is the sum of the individually given numbers.

outcome of a die that they have put bets on, they deliver a number each from 1 to 6 to an administrator. The administrator then sums the individually delivered numbers and consecutively subtracts 6 from the result until a number less than 6 remains. This number plus one constitutes the outcome of the mimicked die.

Among other things, this opens the possibility for a very effective roulette game, where no roulette wheel is needed. Each participant sits in front of an electronic device with 37 buttons, numbered from zero to 36. After they have completed their stakes, they secretely press a freely chosen button. The result is then delivered according to a procedure corresponding to how the outcome of the die was calculated above.

The kind of irregularity represented by the procedure presented in this section can be considered as a kind of disorder that is even more disordered than randomness. This is implied by the fact that there are no reasons to expect that the outcomes of this procedure can be deterministically deduced from the conditions and neither that it will follow the *Law of Stability of Statistical Frequencies* and the *Principle of the Impossibility of Gambling Systems*.[9] The existence of such a type of disorder that is more irregular than randomness is accordance with the conclusions in section 3.

7 Discussion

Randomness is a foundational concept in quantum physics, statistical mechanics and in parts of mathematics that deal with statistics and probabilities as well as in everyday life. Despite its widely spread applicability, there is a lack of consensus about both the characteristics of randomness and its definition. In order to bring some clarity to the subject, a number of misconceptions about randomness have been scrutinised in this essay.

Contrary to the presented misconceptions, the following conclusions have been drawn: (1) randomness does not represents maximal disorder, (2) randomness can be a source of structure and order, (3) randomness has a direction in time, and (4) randomness is not necessarily a requisite for chance events with a fair outcome.

These results give deep insights into the nature of randomness. Of particular importance is the conclusion drawn in section 3 that the characteristics of randomness calls for an explanation, and that this explanation must have the form of some "missing" physical law. This is implied

[9] For example, the preference for the different individually delivered numbers may hypothetically be different for women than for men, and certain numbers can represent good luck at some places, which in both cases can give rise to a lack of translation invariance with regard to the outcomes.

by the fact that no *known* physical law describes under which conditions randomness (and not some even less regular kind of disorder) is at hand in repeatable and disordered events.

A physical law that can fill this void has been proposed by Skoruppa (2022b,c) and describes that law-like future-directed probabilities rather than law-like past-directed probabilities rule physical processes under certain conditions. This law is further justified in the present essay, since it has become clear that randomness has a direction in time, which can give rise to growth of structure and order.

Hopefully, these conclusions can provide some steps towards a deeper understanding of the concept of randomness and its relations to other forms of disorder and irregularities. In this connection the definition of Bunge (1999), presented in section 2 and further elaborated in (Skoruppa, 2022d, chapter 14), may contribute with a map that can facilitate the journey.

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