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Appendix V. Time asymmetry in macroscopic stochastic processes

Abstract

Five macroscopic stochastic phenomena are analysed with respect to the time order of the condition and the outcome of the time translation invariant conditional probabilities that correctly describe the processes. The preparation procedure is discussed as an explanation of the demonstrated time asymmetry, but this hypothesis is found unsatisfactory. Instead, the analysis shows that the time asymmetries are implied by a recently proposed physical law.

1 Introduction

The search for a general description of the time asymmetries of the universe has occupied physicists for a long time. Enumerations of physical time asymmetries include, as a rule, the expansion of universe, the irreversibility of thermodynamics, and radiation from a centre. Five additional time asymmetries have been mentioned occasionally: the gravitational aggregation of matter, radioactive decay, the predictability of quantum processes, the existence of black (but not white) holes, and the CP non-invariance of the weak interactions.[1]

An asymmetry, previously only given negligible attention, is the difference between the past and future time direction in *macroscopic* stochastic processes, i.e. processes where the constituent stochastic events are macroscopic.[2] The present article aims to find a general description of this time asymmetry and to investigate its relation to the *law of statistical time asymmetry*, which is proposed in Skoruppa (2022d).

[1]Enumerations of time asymmetries can be found in Gal-Or (1972), Penrose (1979), Davies & Gribbin (1991), Zeh (1992), Gell-Mann & Hartle (1994), Cramer (2000), Dorato (2000) and Smolin (2013). Doubts have been raised about whether the CP non-invariance of weak interaction is really a *time* asymmetry (Watanabe, 1965; Skoruppa, 2022c).

[2]Two examples of time asymmetric probabilities in macroscopic systems can be found in Salmon (1979) and Arntzenius (1995): a bulb-producing factory and random walking toys, respectively.

To facilitate the investigation, the following analyses will primarily be focused on systems with individual processes having the Markov property. This demarcation is convenient, since it is easier to ascribe empirically verified, lawlike probabilities to individual Markov processes than to stochastic processes that is not having this property.

The outline of the paper is as follows. First, some basic concepts are defined in Section 2 and, in Section 3, macroscopic stochastic processes are divided up in four categories. In Sections 4, 5 and 6, three examples of such processes are described and proven to be time asymmetric with respect to the conditional probabilities describing them. This is found to be a result in accordance with the *law of general statistical time asymmetry*. In Section 7, similar results are demonstrated in the non-Markovian process of mixing. In Section 8, the assumed time asymmetry of preparation is scrutinized, and in Section 9 the *law of general statistical time asymmetry* is additionally verified through a qualitative line of reasoning applied on processes concerning black holes. In Section 10, finally, the conclusions drawn from the findings are summarized.

2 Basic concepts

In the following presentation, the concepts *future-directed probability* and *past-directed probability* are of central importance. They are defined as follows (given that $A[t_1, t_2]$ and $B[t_3, t_4]$ denote arbitrary states of a given system at time intervals $[t_1, t_2]$ and $[t_3, t_4]$, respectively, where $t_1 \leq t_2 \leq t_3 \leq t_4$ and $t_1 < t_4$).

Definition 1 *Future-directed probability* is conditional probability in which the conditional state precedes the outcome state*:*

$$p(B[t_3, t_4]|A[t_1, t_2]) = \frac{p(A[t_1, t_2] \wedge B[t_3, t_4])}{p(A[t_1, t_2])} \ .$$

Definition 2 *Past-directed probability* is conditional probability in which the outcome state precedes the conditional state*:*

$$p(A[t_1, t_2]|B[t_3, t_4]) = \frac{p(A[t_1, t_2] \wedge B[t_3, t_4])}{p(B[t_3, t_4])} \ .$$

Furthermore, in Skoruppa (2022d), a law that describes time asymmetry in physical stochastic processes is proposed:

> **Law of statistical time asymmetry**
> Assume that $A[t_1, t_2]$ and $B[t_3, t_4]$, where $t_1 \leq t_2 \leq t_3 \leq t_4$, are physical states, defined by the same quantities in an isolated process, where only one of the non-deterministic probabilities $p(B[t_3, t_4]|A[t_1, t_2])$ and $p(A[t_1, t_2]|B[t_3, t_4])$ is time translation invariant. Then the former probability is always time translation invariant.

The law thus states that future-directed probabilities have some kind of superiority over past-directed probabilities regarding which one of them that can describe time translation invariant probabilities in an isolated process.

A complication connected with the application of this law to macroscopic systems is that these are usually described as deterministic when isolated. Therefore, a macroscopic process can be correctly described by time translational invariant conditional probabilities with either the value one or the value zero in both time directions. However, there is a category of processes for which the law is applicable, if it is modified by assuming some kind of disordered influence, which corresponds to the "molecular chaos" in statistical mechanics and to the presumed fundamental randomness in quantum mechanics. Such disordered influences is an essential part of most of the mechanisms that bring randomness into games of chance, such as lottery, dice, card games and roulette. Thus, the following generalized version of law above will be investigated in the present article:[3]

> **Law of general statistical time asymmetry**
> Assume that $A[t_1, t_2]$ and $B[t_3, t_4]$, where $t_1 \leq t_2 \leq t_3 \leq t_4$ are physical states, measured by the same quantities in a process that is uncontrolled in the time interval $[t_1, t_4]$, and where only one of the non-deterministic probabilities $p(B[t_3, t_4]|A[t_1, t_2])$ and $p(A[t_1, t_2]|B[t_3, t_4])$ is time translation invariant. Then the former probability is always time translation invariant.

The word "uncontrolled" here means that no manipulation is made during the process to control the outcome of the process in either time direction, which is a more loose condition than isolation.

To analyse *lawlike* time asymmetries in stochastic processes, it must be verified that the processes can be correctly described by probabilities that are invariant under space translation or under time translation, or

[3] A sketchy derivation of this law is provided in Skoruppa (2022e).

both – otherwise the probabilities cannot be verified through repeated experiments. In the present article, the probabilities are assumed to possess both kinds of translation invariance or non of them. *Time translation invariance* therefore also includes space translation invariance in this paper.

Furthermore, the analysis mainly takes *Markov processes* into consideration. Such processes are assumed to have "no memory" and "no foresight",[4] which implies that the conditional transition probabilities governing the processes do not depend on any other state than the conditional state that is temporally closest in either the past or the future time direction.[5] A special case of the Markov process is the *Bernoulli process*, which does not depend on any history at all, and where the probability of every state therefore is independent of all other states.

3 Four categories of processes

Stochastic processes, generally, can be categorized as individual or composite events. A single Markov process, in its turn, can be analysed with respect to whether it is *continuous-time* or *discrete-time*, i.e. whether the probabilities measure the likelihood of events in continuous time or not. Since most games of chance use Bernoulli processes as the basis for their stochastic devices, this kind of process represents the basis for the discrete-time processes in the present article. All processes are assumed to be *homogeneous*, i.e. they are possible to correctly describe with time translation invariant probabilities, in at least one time direction. This amounts to four classes of stochastic macroscopic processes:

1. individual discrete-time Bernoulli processes,
2. ensembles of processes of category 1,
3. individual continuous-time Markov processes, and
4. ensembles of processes of category 3.

The first category can be found, for example, in games of chance, where one of a number of alternative outcomes is realized by a single object or device. Individual, discrete-time Bernoulli processes can also appear in macroscopic systems in nature, such as a floating twig following one of several down-stream branches of a river or the orientation of

[4]For a definition of the Markov property that is neutral with respect to time direction, see Uffink (2007), who also describes the Bernoulli process.

[5]There are other macroscopic stochastic processes that show time asymmetric evolutions – for example mixing or the formation of traces – but they are more complex and therefore more difficult to describe mathematically. Nevertheless, mixing is described in Section 7.

a needle falling from a tree (if, instead, the moment of the falling of the needle is the stochastic variable, the process is continuous-time). This type of process is characterized by single events, described by probabilities that are not related to the time interval between the conditional and the outcome state. For example, a die comes to rest with equal probability for each of the six possible sides upwards, irrespective of how long time the roll takes (if it is not too short); a fallen needle can be assumed to have the same probability to be oriented north-south as east-west, irrespective of the time it takes to fall.

The second category of macroscopic stochastic events appears when multiple events of the first category are summed up. Examples are the total number of spots after a throw with more than one die, or the number of playing cards in sequence after the deck is thorough shuffled. The present analysis is concerned with composite processes that can be in a state of equilibrium. In such processes, a type of explicit time-dependence appears, where the probability of the final outcome depends on how many members of the ensemble that are in a given state, given the initial conditional state. A familiar example is the gradually increasing disorder of a new pack of cards that is repeatedly shuffled. Another example of such a process, analysed in Section 5, is a box containing one hundred coins, only some of which are turned over when the box is shaken.

Examples of the third and fourth categories are more difficult to find in nature. Occurrences of earthquakes of a given magnitude or occurrences of shifts in quasar light intensity have some empirical support for random distribution in time.[6] However, the individual events that build these phenomena up do not have the Markov property, since they tend to appear in groups. A macroscopic process that might posses the Markov property is raindrops falling on a windowsill, for which the time-intervals can be assumed to be randomly distributed.

It is also possible to create artificial macroscopic continuous-time Markov processes. For example, a ball can be shaken in an irregularly shaped box with a hole in the bottom, after which the time until the ball leaves the box is measured. In Section 6 an experiment with similar properties is described: a ball placed in an unstable position on a jet of water.

4 The die under the cup

There are many well-known macroscopic processes in which a single random event displays multiple possible outcomes with probabilities that neither depend on what happened before the event nor the length of

[6]Johnston & Nava (1985); Press (1978).

the time interval between the condition and outcome states. Some of these *discrete-time* Markov processes can be found in games of chance, where a die, a lotto machine, a roulette wheel, or a well shuffled pack of cards can provide different numbers of randomly distributed outcomes. Comparable games can also be played by throwing an arbitrary object that can come to rest in a variety of positions, for example a match box, a drawing pin, or just an ordinary stone. Some people pick petals from a flower to find out whether the giver loves them, and others divide a pile of yarrow stems to reveal what the Chinese oracle book *I Ching* tells about the future. In nature, the orientation of a falling pine needle or a meteorite hitting an arbitrary longitude can be considered such an individual discrete-time Bernoulli processes.

If these processes are repeated, the relative frequencies of the *final* states will tend to stabilize more and more (with the possible exception of the petal picking procedure). In a game of chance, the resulting relative frequencies can also be derived from the symmetric properties of the device in question.[7] In contrast, the relative frequencies of the *initial* states do not necessarily stabilize as a result of accumulated observations. The initial state is determined by factors lying outside the process of the random event.

In all these processes, some kind of random producing event *precedes* the random outcome in question. It is the position of the die, the drawing pin, or the stone *after* the throw that is assumed to be ruled by the laws of chance, not the position before the throw. In a similar manner, it is the state of the roulette ball, the lotto machine, or the yarrow stems *after*, not before, a given chance event that is intended to induce randomness.

To find out whether this apparent asymmetry of time is lawlike, it is suitable to perform experiments in which the initial and final boundary conditions are as equal as possible. The aim of this time symmetry of the experimental setup is to minimize the possibility that the *experimental procedure* is the source of the asymmetry. A simple experiment can be carried out in the following way.

(i) A die is placed on a table.

(ii) A cup is placed upside down on top of the die.

(iii) The cup is shaken vigorously for a couple of seconds in the time interval $[t_1, t_2]$, making the die bounce inside the cup.

(iv) The cup is lifted from the die.

(v) The die is taken from the table.

[7] Strevens (1998).

In agreement with the ambition to keep the initial and end states as equal as possible, this course of events is kept symmetric in time in every significant respect. A film recording of the process will look essentially the same, irrespective of whether the film is played forwards or backwards. Thus, the experiment outwardly shows a degree of time symmetry, which is as high as the laws of physics allow. Friction, heat dissipation, and absorption of light imply that time asymmetric processes occur, but, reasonably, these processes do not influence the probabilities of the outcome.

A crucial question now is whether time symmetry also prevails for the probabilities that correctly describe the event. The die is assumed to be well balanced, hence the following probabilities hold:

$$p(\text{die shows one spot } [t_2] | \text{arbitrary state } [t_1]) = \frac{1}{6} ,$$
$$p(\text{die shows two spots } [t_2] | \text{arbitrary state } [t_1]) = \frac{1}{6} ,$$

and so forth.

These probabilities are future-directed since the condition precedes the outcome. If the procedure is repeated over and over, the relative frequencies will stabilize around $\frac{1}{6}$, without any systematic pattern appearing. The probability can also be derived from the physical properties of the die, since the six sides are symmetric in relation to each other, and since the centre of gravity of the die is assumed to be at its midpoint.

On the other hand, it is not possible to specify any time translation invariant values of the past-directed counterparts of the probabilities above:

$$p(\text{die shows one spot } [t_1] | \text{arbitrary state } [t_2]) ,$$
$$p(\text{die shows two spots } [t_1] | \text{arbitrary state } [t_2]) ,$$

and so forth.

For example, a person or a machine could always place the die with six spots upwards at t_1, or always let an odd number of spots constitute the state at t_1.

The possible influence of a person or a machine implies that the unconditional probability of the state of the die before the throw can vary, and this possibility can influence the past-directed but not the future-directed probabilities. The time asymmetry, which becomes apparent in such cases, therefore is in accordance with the *law of general statistical time asymmetry* presented in Section 2, which states that if only one kind of time-directed probability is time translation invariant, it is the future-directed probabilities that have this feature, whereas the past-directed probabilities do not.

The difference between the future- and past-directed probabilities above may seem, at first glance, to be an unavoidable consequence of the experimenter's ability to establish the initial state in contrast to the final state. This difference, in its turn, can appear to be an effect of a human agent's ability to plan the preparation of the experiment by using memory. However, there can also be a common cause behind both the time-directed probabilities and the human memory. The ability to remember the past – and thereby the difference between the human abilities to prepare and "postpare" an experiment – could be an effect of a universal lawlike time asymmetry, expressing itself through the fact that future-directed probabilities rather than past-directed probabilities are time translation invariant. The role of preparation is further examined in Section 8.

One of the reasons for the difficulties in discriminating between these two interpretations of the die-under-the-cup experiment is the intrinsic time asymmetry arising from the fact that the state of the die before the throw *can* be restricted by human decisions, while the state after the throw cannot. To decide which description – time asymmetry implied by the human agent or by a physical law – is correct, it is desirable to create a macroscopic stochastic process, for which both the initial and the final states are restricted. Hence, the experiment to be analysed in Section 5 has this property.

5 The coins in the pizza box

Discrete-time macroscopic Markov processes are quite rare in natural settings. Processes consisting of a number of such events, which therefore can proceed towards a state of equilibrium, are consequently even more rare. However, they can be constructed by assembling multiple processes of the kind described in the last section.

A process with such properties, with respect to the symmetry of the boundary conditions, will be studied in this section. It resembles a well-known model of a gas system, introduced by Paul and Tatiana Ehrenfest,[8] and it evolves in the following way.

(i) 100 coins are situated in a pizza box (or something similar), with a recorded number between 70 and 80 lying heads upwards.

(ii) The box is closed.

(iii) The box is shaken in a way that makes 10 of the 100 coins to turn on the other side.[9]

[8]Ehrenfest & Ehrenfest (1907).

[9]Since this is a thought experiment, the number of coins that turns can be assumed

(iv) The box is opened.

(v) The 100 coins are situated in the box and a number between 60 and 90 is registered lying heads upwards.

The description of the evolution of this process differs from the description of the die in the cup in three essential ways:

- While the state of the die under the cup is described with respect to the position of a *single* object, the state of the coins in the pizza box is described with respect to the *distribution* of positions.
- While the state as a rule is *completely* changed between two shakings of the die under the cup, the state is only *partially* changed as a result of the shaking of the pizza box.
- While the probabilities of the two consecutive states of the die, shaken under the cup, are *independent*, the probabilities of the two consecutive states of the coins in the pizza box are *dependent* on each other.

The conditions above imply that there is a possible *equilibrium* state for the coins in the pizza box in the form of the distribution of 50 heads and 50 tails. This equilibrium state is characterized by a greater number of unique states of single coins corresponding to the equal distribution than to any other distribution. The second condition implies that the evolution of the process has an inertia, which is demonstrated by the limitation placed on a given state by the distribution of the other state (with shaking in between), which is highlighted by the third condition.

Again the experiment is constructed with the aim that the conditions of the process should be time symmetric. A film recording of the procedure (i)-(ii)-(iii)-(iv)-(v) above will look essentially the same if it were played backwards, i.e. in the time order (v)-(iv)-(iii)-(ii)-(i). The boundary conditions, therefore, can be regarded as time symmetric (heat spreading, friction and other thermodynamic processes that are expected not to influence the evolution of the experiment with respect to heads and tails are disregarded).

This shaking process gives rise to randomness, in the sense that each coin in the pizza box is assumed to have an equal probability to be turned over. This, at first glance, may appear to be a perfectly time symmetric supposition, but in practice it is not. As a matter of fact, such an assumption of equal probabilities gives different results depending on whether the initial (i) or the final (v) state is considered to be the conditional state. To demonstrate this, three mutually exclusive hypotheses can be formulated.

to be fixed. In a realistic experiment, the number will rather be fluctuating, but this would not change the essential conclusions drawn on the basis of the experiment.

A. Since it is equally probable for each coin in the *initial* state (i) to be turned over, and since there are more heads than tails in the initial state (i), more coins is probably turned from heads to tails than the reverse, during the shaking procedure (iii); therefore, the proportion of tails is expected to be larger in the final state (v) than in the initial state.

B. Since the equal probability for each coin to be turned over during the shaking procedure (iii) is *not intrinsically time asymmetric* and therefore should imply an expected time symmetry, the expected number of coins that is turned from heads to tails equals the expected number of coins that is turned from tails to heads; therefore, the expected numbers of heads and tails remain constant from the initial state (i) to the final state (v).

C. Since it is equally probable for each coin in the *final* state (v) to be turned over, and since there are more heads than tails in the final state (v), more coins is probably turned from tails to heads than the reverse, during the shaking procedure (iii); therefore, the proportion of tails is expected to be larger in the initial state (i) than in the final state.

Assume that the ten coins that are turned over are randomly chosen from the total number of coins. Then, these hypotheses can be expressed mathematically as probabilities of an individual coin to be turned over, where t_1 is the time of the initial state and t_2 is the time of the final state:

$$\text{(A)}: p(\text{heads } [t_2]|\text{tails } [t_1]) = p(\text{tails } [t_2]|\text{heads } [t_1]) = \tfrac{1}{10}\ ; \qquad (1)$$

$$\text{(B)}: p(\text{heads } [t_1] \wedge \text{tails } [t_2]) = p(\text{tails } [t_1] \wedge \text{heads } [t_2]); \qquad (2)$$

$$\text{(C)}: p(\text{heads } [t_1]|\text{tails } [t_2]) = p(\text{tails } [t_1]|\text{heads } [t_2]) = \tfrac{1}{10}\ . \qquad (3)$$

According to the definitions in Section 2, equation (1) represents an equality between two future-directed probabilities, while equation (3) represents an equality between two past-directed probabilities. It is evident that equation (3) is the time reversed counterpart of equation (1), since the only difference between the two non-numeric expressions is that t_1 and t_2 have changed places. Equation (2), on the other hand, represents an equality between two joint probabilities without direction in time; their value is derived from the fact that the sum of these probabilities is $\frac{1}{10}$.

The reasons, given in hypotheses A, B, and C, are all logically consistent and in accordance with the prevalent physical laws. Therefore, none of the three probabilities (A), (B), and (C) is more valid than the other as a result of deductive reasoning. Consequently, the question of which one is correct has to be settled empirically.

Nevertheless, there is actually one theoretical reason to prefer hypothesis A to hypothesis C: the *law of general statistical time asymmetry.* The proposed law states that, where either the future-directed or the past-directed probability of the stochastic process is time translation invariant, it is the former type of probability that has this feature. This means that the law predicts that hypothesis A can correctly describe the process if there exist any time translation invariant conditional probabilities, while it rejects hypothesis C. If hypothesis B is correct, the expected state of the process is stationary and both the future-directed and the past-directed probabilities are time translation invariant. Therefore, hypothesis B is not affected by the law.

As many people would guess, hypothesis A also turns out to fit best with empirical facts. In a long series of experiments the relative frequencies will be distributed in accordance with probability (1).

Consequently, it can be concluded that in a macroscopic experiment, where both the initial and the final states are restricted, there can still exist a time asymmetric evolution. Thus, the experimenter's capacity to restrict the initial state cannot be the general mechanism behind macroscopic time asymmetry. Instead, the *law of general statistical time asymmetry* describes an evolution in accordance with empirical facts. The role of preparation is scrutinized in Section 8 as a possible explanation, but before that, some more empirical findings are investigated in the following two sections.

6 The ball on the jet

The difficulty to find explicitly continuous-time macroscopic Markov processes, possible to describe correctly by probabilities that are time translation invariant, is mentioned in Section 3. Earthquakes and quasar light were referred to, but they have a tendency to appear in groups, which makes it difficult to ascribe well-defined time translation invariant probabilities to the individual events. Even water-drops on a window-sill can be assumed to come at a varying expected rate as a result of phenomena such as cloud formation and wind. It therefore seems to be problematic to demonstrate time translation invariant probabilities for most continuous-time macroscopic Markov processes.

As a part of the work reported here, a series of experiments was made to provide an empirical example of a continuous-time macroscopic Markov process. A table tennis ball was placed to balance on an upwards directed jet of water, and the angle of the jet from the horizontal plane was adjusted to make the system unstable. It was hypothesized that the turbulence of the water stream would produce a behaviour of the system in accordance with a continuous-time Markov process in a similar way as the decay of radioactive atoms.

The intervals were recorded in a series of 50 trials, each one measuring the time interval from the instant when the ball was placed on the jet to the instant when it fell off.[10] These intervals were found to be distributed in accordance with what, according to Skoruppa (2022a), is expected from a series of continuous-time Markov processes with probabilities that are time translation invariant ($[t_0]$ denotes an arbitrary point in time when the time measurement starts; $\Delta t > 0$):

$$p(\text{ball on jet }[t_0 + \Delta t]|\text{ball on jet }[t_0]) \approx e^{-\frac{\Delta t}{8.6s}}\ , \tag{4}$$

where 8.6 seconds is the average time before the ball falls off the jet. This probability is future-directed, according to Definition 1, and it seems reasonable to assume that it also is time translation invariant.

The question now is, if the experiment with the ball on a jet of water is a validation of the *law of general statistical time asymmetry*. Actually, a modification of the argument given in Section 4 is suitable to show that also this process obeys the law. The crucial difference is that while the die in the cup has different outcomes with respect to the number of dots, the ball on the jet has different outcomes with respect to time itself.

If the trials of the experiment with the ball are initiated at randomly chosen points of time, the experiment is perfectly time symmetric, and both past-directed and future-directed probabilities that are time translation invariant may seem to correctly describe the process. The ball is placed on the jet at a random point of time and it falls of at a random point of time. However, it is only the randomness that characterizes the point of time of departure from the jet that is *guaranteed*.

In a similar way as in the experiment with the die in the cup, an experimenter or a machine can start the experiment with the ball on the jet in a systematic way. For example, the ball can always be placed on the jet when a certain digital clock shows exact minutes, or a series of trials can be administrated where one hundred balls are placed on one hundred jets of water at more or less the same time.[11] In both cases there will be a systematic pattern of the outcomes with respect to the initial state. Since the past-directed probability conditions do not exclude these possibilities, the past-directed probabilities in the experiment with the

[10] The following time intervals were recorded in the somewhat primitive experiment (seconds): 0, 22, 0, 13, 8, 20, 18, 4, 0, 5, 25, 13, 18, 4, 19, 3, 10, 33, 5, 4, 0, 4, 8, 7, 3, 0, 1, 5, 11, 5, 15, 7, 20, 7, 8, 1, 5, 8, 1, 1, 2, 9, 25, 19, 5, 2, 4, 14, 9, 1.

[11] The second case is close to what can be observed in samples of radioactive decay atoms, where a great number of atoms have been created at more or less the same time (usually in stars).

ball on the jet are not necessarily time translation invariant. I.e. the probability

$$p(\text{ball on jet } [t_1 - \Delta t] | \text{ball on jet } [t_1])$$

is not independent of t_1. In such cases, the *law of general statistical time asymmetry* is validated, since the future-directed probabilities are always time translation invariant in this process.

It is also possible to construct a model with macroscopic objects that mimic the molecules in an expanding gas. A previous author has suggested an experiment, where some balls are placed in a rectangular tray, divided in two equal areas with a small opening between them, and where the tray is constantly shaken.[12] This system can be regarded as a large scale model of a system of gas molecules and their heat motions, where an initially unequal distribution evolves towards equilibrium. The behaviour of the balls in the tray is actually a composite process, built up by a continuous-time macroscopic Markov process, since every ball can be considered an individual system. As in the case of the ball on the water jet, this system can be correctly described by future-directed time translation invariant probabilities in accordance with the *law of general statistical time asymmetry.*

7 Statistic time asymmetry in the non-Markovian macroscopic process of mixing

The examples given in the preceding sections may seem as rare examples of limited empirical value. However, the support for the *law of general statistical time asymmetry* in these macroscopic Markov processes can be considered as a support for the application of this law to macroscopic processes in general. This hypothesis will be tested in the present section by applying the *law of general statistical time asymmetry* to macroscopic processes that do *not* posses the Markov property. If the consequences of the application are in accordance with empirical facts, the hypothesis is verified.

The processes in question are different kinds of *mixing* of macroscopic constituents, for example clothes tumbling in a washing-machine, cereal and fruit pieces gathered before being poured into a package of muesli, or sugar and cocoa powder being prepared for hot chocolate.[13] In nature, other examples of such processes are mixing of different kinds of stone grains during forming of gravel and leaves of different sorts that are blowing in the wind.

[12]Watanabe (1966).

[13]Arntzenius (1997) discusses the time asymmetry of other examples of non-Markovian macroscopic processes: peg-boards, "strings in the wind" and breaking metal beams.

The following schema is a general description of a mixing process:

1. At least two types of particles or objects are gathered in space but separate with respect to type.

2. The particles or objects move chaotically with respect to each other.

3. The (at least two) types of particles or objects in question are gathered in space but are not separate with respect to type.

The time asymmetry of the phenomenon is obvious since mixing processes represented by the order 1-2-3 are quite usually seen, while "unmixing" processes represented by the order 3-2-1 are seldomly seen.[14]

It may be tempting to apply the thermodynamic concept of entropy to this kind of process, and attempts in this direction have been done recently.[15] However, the domain of applicability of thermodynamics is restricted to processes that evolve *spontaneously* in *isolated* systems, while macroscopic mixing requires energy from the environment in order to provide some kind of stirring, tumbling or the like.

Instead, the *law of general statistical time asymmetry* is applicable to this kind of systems, since the process is "uncontrolled" in stage 2 in the schematic description above. Now, the consequences of applying this law to the general mixing process will be studied in a qualitative and admittedly sketchy analysis. The line of reasoning is based on the fact that there are – with some arbitrary degree of coarse graining – more distributions of constituents representing a state with higher degree of mixing than there are distributions representing a state with lower degree of mixing (an example of how this can be measured is presented in the case of the deck of cards below).[16] As a consequence, it is reasonable

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[14]Spontaneous unmixing processes are seen on molecular level in spontaneously separating liquids, where the molecules move chaotically due to motion of heat (for example water and oil). Evolutions towards partially ordered states can also be seen in some macroscopic systems, e.g. the gathering of large muesli particles near the top of the package.

[15]Carroll (2010) writes:

> Without (yet) being precise about the mathematical definition of 'entropy', the example of blending two kinds of colored sand illustrates why it is easier to mix thing than to unmix them. [...] The reason is simple: To separate out of two kinds of sand that are mixed together requires a much more precise operation than simply shaking or stirring. (Carroll, 2010, p. 144–145)

This "simple" explanation, however, leaves the question unanswered why a "precise operation" is required to unmix grains of sand, while no such action is required to mix them.

[16]Different methods for describing degree of mixing are summarized by Wen et al. (2015).

to conclude that there are more assumed equally probable paths of evolution (with regard to the possible motion directions of the constituents) that connect a partially mixed state with a state with a higher degree of mixing than with a state with lower degree of mixing.[17]

Assume that the future-directed probabilities are time translation invariant according to the *law of general statistical time asymmetry.* Moreover, assume that the conditional state is partially mixed. Then, the probability of a *future* state with a *higher* degree of mixing given this condition is higher than the probability of a *future* state with a *lower* degree of mixing given this condition. This follows from the greater number of (approximately equally probable) paths leading to future states with a higher degree of mixing. Thus, the most probable evolution, according to the *law of general statistical time asymmetry*, is that a partially mixed state evolves towards higher degrees of mixing in the future.

On the other hand, if a partially mixed system is assumed to evolve according to *invariant past-directed probabilities* applied to the equally probable paths of evolution, the most probable evolution would go in the opposite direction. In this case, the probability of a *past* state with a *higher* degree of mixing, given the condition of a partially mixed state, is higher than the probability of a *past* state with a *lower* degree of mixing, given the condition of a partially mixed state. This follows from the greater number of (approximately equally probable) paths leading from past states with a higher degree of mixing. Therefore, the most probable evolution, according to time translation invariant past-directed probabilities, is that a partially mixed state evolves towards *lower* degrees of mixing in the future. Since empirical findings indicate that large-scale mixing processes evolve towards higher degrees of mixing, i.e. according to what is stipulated by the *law of general statistical time asymmetry*, these processes imply a verification of the law.

This verification of the *law of general statistical time asymmetry* can easily be quantified in simple models of mixing. A one-dimensional mixing process can be demonstrated with a deck of cards, in which a limited number of randomly chosen pairs of cards changes place during each step of the evolution (which is a Markov process modeling the generally non-Markovian mixing process). A deck is initially ordered in sequenced suits and then, step by step, is shuffled in the following way. The deck is cut, whereupon the order of the two cards on the top are switched. If this is repeated many times in succession, the suits will probably become more and more thoroughly mixed in the future than they were in the past.

[17]This assumption is in the vein of Tolman (1938, p. 59) who formulated a hypothesis of "equal *a priori* probabilities for equal regions in phase space".

The process can be considered time symmetric with regard to the initial and final boundary conditions of the shuffling procedure, since it would be difficult to tell if a film recording of this process is played backwards or forwards. Still the suits in the pack of cards can be expected to become gradually more shuffled. The degree of mixing is suitably measured by the number of neighbour cards having the same colour, and the consequences of translation invariant past-directed and future-directed probabilities are studied with a partially mixed state as the conditional state. The time translation invariant future-directed probabilities then will prove to describe the same kind of evolution towards a higher degree of mixing that is usually observed as a result of ordinary mixing of a deck of cards, while time translation invariant past-directed probabilities cannot do this. Thus, the law is verified.

So far, the *law of general statistical time asymmetry* seems to be verified for the macroscopic processes analysed in the present article. However, the role of the human intervention in most of these processes motivates a closer examination of the preparation process, which is the subject of the next section.

8 The time asymmetry of preparation

Four kinds of macroscopic phenomena have been described according to the *law of general statistical time asymmetry* in the previous sections, and the law is apparently verified in these processes. However, an alternative explanation for various time asymmetries has been put forward by previous authors,[18] namely the human ability to prepare an experiment, while the ability to take the time reversed action – what is here called "postpare" – is lacking.

On an intuitive level, this time asymmetry of experiments in physics may seem self-evident, but on closer inspection it is difficult to formulate in precise terms. Normally an experimenter intervenes with an experimental system that is supposed to be ruled by the laws of physics during the experiment, without any further disturbance from the environment. Since a human being can remember the past and not the future, she or he usually builds experimental arrangements that are ready for use at the future end of the intervention. Hence, a planned and deliberate intervention usually precedes the experiment and, consequently, human beings experience that they are able to affect the initial but not the final end of the intervention.

However, the difference from the experimenter's point of view, with respect to the initial and the final states of an experimental process, does not seem to have any significance in the realm of physics. The system is

[18] For example, Watanabe (1966), Leggett (1977) and Bohm et al. (1994).

isolated, or at least partly isolated, from the start to the end of the experiment, and both the start and the end consist of an intervention with the system. What happens in the mind of the experimenter does not influence the physical description of the process and, thus, there is nothing that can be interpreted as a physical time asymmetry of preparation. The only thing that could achieve such time asymmetry is the possibility that the experimenter prepares initial states that are qualitatively different from the final states. However, this hypothesis is contradicted in the case of the experiment with the coins in the pizza-box in Section 5, where the initial and the final states in essence have the same characteristics, and where a time asymmetric evolution still exists.

A way to find out whether experimenters really cause macroscopic time asymmetry is to remove all human influence from macroscopic statistical time asymmetry phenomena, to see whether the time asymmetry still is there. Unfortunately, this is nearly impossible, since the random element in macroscopic time asymmetric processes mostly are the result of some kind of human intervention (possible exceptions are found, e.g. in mixing of leaves by the wind). Therefore this is not an appropriate test. A better clue is given by the fact that there are microscopic phenomena that develop according to the *law of statistical time asymmetry*, without intervention from any human experimenter: for example, radioactive decay, expanding gas, chemical reactions, and radiation from a centre.[19] This radically weakens the hypothesis that statistical time asymmetry is caused by a human intervener.

The most fundamental objection to the preparation hypothesis is that the influence of the experimenter's act of planning is not an issue of physics. It does not seem possible to falsify the proposal that the time asymmetries described in the previous sections are consequences of the human intervention; accordingly, the influence of preparation is strictly speaking a metaphysical issue in these cases. Hence, there is no place for preparation as an explanation for the macroscopic statistical time asymmetry in the realm of physics.

Lawrence Sklar expresses a similar point of view:

> 'Preparation' is used frequently in attempts to rationalize statistical mechanics, but it is a slippery and difficult notion. If it means just selecting for our consideration a collection of systems, it is hard to see how one can justify the claim that initial ensembles are prepared but final ones are not. If it is, instead, a reference to the physical process by which the systems are brought to their states, the initial ones by being 'prepared' and the final by 'evolving from prepared

[19] These processes are demonstrated to be verifications of the *law of statistical time asymmetry* in Skoruppa (2022b,c,f).

> states', it is hard to see how the time asymmetry here is not being smuggled into the rationalization by a reliance on some notion of time asymmetric causation which allows the derivation of the idea that in the evolution of systems from one time to another the 'preparation' of the system is always in the past time direction from its 'measurement'. (Sklar, 1986, p. 220)

Until the difference between preparation and 'post-paration' is given a clear-cut physical description, the concept of preparation has no place in a physical theory about statistical time asymmetry. Instead, the *law of general statistical time asymmetry* gives the best available description of the probabilities in Sections 4–7.

9 The time asymmetry of black holes

An extremely large-scale example of macroscopic time asymmetry is represented by the fact that there exist *black holes* but no time reversed version of these, conventionally named *white holes*, in the universe.[20] A black hole swallows up everything that comes near to it – e.g. planets, stars, galaxies and light – and it is only expected to emit very small particles.[21] White holes, on the other hand, would be spewing forth large-scale matter, but no one has ever seen any sign of such celestial bodies.

If this time asymmetry is to be directly described by the *law of general statistical time asymmetry* from Section 2, it would be necessary to quantify the probability that a planet or a star is swallowed or spewed forth by a black or white hole at a certain point in time. This is, however, not possible, since there is not enough empirical ground for the required calculations. Instead, a qualitative line of reasoning has to be applied in order to explore *indirect* signs that show if the black hole time asymmetry is in harmony with the mentioned law.

In this connection, the swallowing and spewing processes of the holes are assumed to be series of individual discrete-time Bernoulli processes. Matter is supposed to move in a random fashion through empty space and, by chance, a certain piece of matter some time in its history is assumed to be so close to a black or white hole that it either have to be swallowed or must have been spewed forth by the hole. This is a discrete-time process when described in terms of probability.

Moreover, the velocity directions of the pieces of matter that move freely in space are assumed to be totally random, both in the distant

[20] Penrose (1979) describes this asymmetry and suggests a "cosmic censorship" of unknown origin that forbids white holes.

[21] Hawking (1975).

past and in the distant future. In other words, these pieces are assumed to have no preferred direction with respect to the localization of any black or white hole.

The next step is to test which kind of time-directed probabilities that correctly describe the evolution of the black holes. First, the interaction between a hole and the surrounding matter is assumed to be a process ruled by time translation invariant future-directed probabilities. The condition is a hole, surrounded by pieces of matter that move randomly through space, and the two possible outcomes are that a piece of matter either has its future inside the hole or that it has not. Since there is a lot of matter of different sizes moving in space, it is probable that some of the pieces, once in a while, find their ways into the hole. This is in accordance with the expected behaviour of the black holes that inhabit our universe.

Second, the interaction between a hole and the surrounding matter is assumed to be ruled by time translation invariant past-directed probabilities. The condition still is a hole with nearby pieces of matter that move randomly through space, but the two possible outcomes are that a piece of matter has its *past history* either inside or outside the hole. Since there is a lot of matter of different sizes moving in space, it is probable that some of the pieces have been *thrown out* of the hole. However, this kind of matter-spewing from a (white) hole is contrary to what is expected from celestial bodies in the universe. Since the conclusion of a matter-spewing hole is based on the implicit assumption of time translation invariant probabilities, this assumption is not confirmed.

In summary, a qualitative line of reasoning, based on the assumptions of time translation invariant future-directed probabilities and randomly moving matter, describes the expected behaviour of black holes, while a line of reasoning, based on the assumptions of time translation invariant past-directed probabilities and randomly moving matter, describes non-existent white holes. It is of crucial importance to recognize that these two line of reasoning are absolutely equal, except for their orientation in time. Thus, the existence of black holes and the non-existence of white holes are in accordance with time translation invariant future-directed probabilities but not with time translation invariant past-directed probabilities. Consequently, the result is also in accordance with *law of general statistical time asymmetry*, since it says that if only one of these two kinds of time translation invariant time-directed probabilities correctly describes a process, it is the future-directed kind.

10 Conclusions

It is demonstrated through various experiments that both individual and composite evolutions of a macroscopic stochastic process can be time asymmetric. Both kinds of processes can always be correctly described with future-directed probabilities that are time translation invariant, whereas it is not generally possible to describe them with paste-directed probabilities that are time translation invariant. Moreover, certain composite macroscopic systems demonstrate a tendency to quantitatively evolve closer to equilibrium, if the system has both its initial state and its final state far enough from equilibrium to make it impossible to reach it during the experiment. Furthermore, similar results are demonstrated in the non-Markovian processes of mixing and the evolution of black holes. Thus, these macroscopic processes evolves in accordance with the *law of general statistical time asymmetry*, which can be considered a general version of the previously proposed *law of statistical time asymmetry.*

Although the intervention of the experimenter seems at first sight to be a possible explanation of the time asymmetry of the processes analysed, this hypothesis could not be verified. The time asymmetry of preparation has no demonstrable physical significance, and the macroscopic time asymmetric processes are therefore better described by the proposed law.

There are notable similarities between some composite, macroscopic stochastic systems and time asymmetric microscopic processes, such as radioactive decay and expanding gas, both of which are microscopic and continuous-time Markov processes. Furthermore, mixing is a phenomenon which can be observed both involving microscopic and macroscopic constituents. Consequently, the size of the system and its individual subsystems does not seem to influence the validity of the *law of general statistical time asymmetry*, which in different versions properly describes stochastic time asymmetry at both microscopic and macroscopic levels.

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